

Properties, Generations and Masses

R Delbourgo¹

*School of Mathematics and Physics, University of Tasmania, Locked Bag 37
GPO Hobart, AUSTRALIA 7001*

Abstract

Schemes based on anticommuting scalar coordinates, corresponding to properties, lead to generations of particles very naturally. In contrast to the standard model, where masses arise through independent Yukawa couplings to a single Higgs isodoublet, property models produce Higgs fields in various multiplet denominations, but contain a single Yukawa coupling. A renormalizable superHiggs potential of quartic order then produces very strongly constrained masses for generations of fields, which depend on just three constants. By allowing for a small parameter, which is meant to characterise the quantum loop effects for the effective potential, one can obtain nonzero masses for the engendered masses. We illustrate the phenomenon for two and three complex coordinates; the more realistic case of five complex coordinates is not yet fully treatable because of its complexity.

PACS: 11.10Kk, 11.30.Hv, 11.30.Pb, 12.10.-g

1 Masses for generations

Two of the most mysterious problems in particle physics are the occurrence of generations (like the electron, muon and tauon) and the manner by which their masses are produced. These problems persist in the standard model where there is no need for generations, or even a limit on them, and where the arbitrariness of the several Yukawa couplings of source fields to the (usually single) Higgs boson is not constrained. Flavour mixings between families in mass eigenstates are yet another source of puzzlement, whether for neutrinos or down-quarks. It is therefore not surprising that much research has gone into seeking simple models which reduce these ambiguities. Supersymmetry, based on spinorial coordinates, is one popular choice but this comes

¹Bob.Delbourgo@utas.edu.au

at the expense of at least doubling the number of particles and relegating the unseen superpartners into unexplored, higher mass regions; other lines of research include grand unified groups, non-commuting spacetime coordinates, superstrings, branes, etc. Given that none of these scheme has received any *direct* experimental endorsement, it is not unreasonable to look for other avenues which may shed light on these conundrums.

Over the last few years [1, 2, 3] we have advocated using anticommuting Lorentz scalar coordinates (ζ) as signposts of particle properties and extending space-time coordinates (x) with these, so that fields become functions both of location and property: $X = (x, \zeta)$. An expansion of a superfield in x and ζ designates where the particle is and what it is. [The construction must be done carefully [4, 5, 6] so as not to conflict with the spin-statistics theorem.] Having just a few independent ζ then leads naturally to a number of particle generations, without having to invoke some external family group. Previous investigations have shown that such a scheme based on 5 complex ζ can accommodate the known fundamental particle spectrum and its repetitions, but they have not yet shown how the masses of the multiplets arise. That is the purpose of the present paper. We will demonstrate that a renormalizable quartic superHiggs potential having only three constants can lead to masses of the sources and scalars which are strongly tied to one another at the semiclassical level. This makes it a good scheme, in that it lends itself to falsification. The more significant point about the present work is that expansion of the superHiggs field $\Phi(X)$ into powers of ζ gives rise to several Higgs multiplets, a few of which can have zero-charge expectation values; but there is only a *single* Yukawa coupling to the superfield source field $\Psi(X)$, so all the masses are fixed by just a few parameters. Of course it is still unclear how quantum loop corrections can alter the results, but as one is dealing with a renormalizable model those effects are in principle controllable or at least can be related to one another. We shall introduce a small parameter scale Δ to account in a crude way for the loop terms in the effective action; otherwise one simply ends up with zero masses, which is unsatisfactory.

We start by studying a purely leptonic model having but two independent (complex) properties ζ . This scheme has only one lepton generation, upon imposing self-duality constraints on the Φ, Ψ expansions in powers of ζ . With such a model the Higgs fields are somewhat unrealistic, being weak isoscalar and isovector; nonetheless the two fermions can in principle be separated in mass scale and they are strongly tied to the two Higgs field masses. In the next section we investigate a more realistic scenario by attaching a third

(colourless) baryonic coordinate, whereupon the superHiggs expansion yields a weak isodoublet Higgs field plus three isocalars and one isovector. This now leads to two generations of leptons and baryons, but these are fixed by the several Higgs expectation values which are approximately determined by the minimisation of the semiclassical potential, allowing for small corrections Δ due to quantum effects. The resulting masses give hope that the more realistic scenario, based on five properties, may be able to account for the known masses and mixings of the three established generations and also escape the experimental exclusion of the single Higgs boson of the standard model. But that is a much more difficult problem which will require the use of and automation by mathematical packages as well as detailed analysis.

2 A purely leptonic scheme

Before getting embroiled with a pair of ζ , we refer the reader to the appendix, where the case of only one complex ζ is treated. There is a simplifying lesson to be learnt there if one is concerned with left and right field components and their gauging. The result of that analysis, so far as the interaction with the Higgs superfield is concerned, is that we can, without error, take the fermion source superfield Ψ to be a Dirac field (left + right) and a function of $\bar{\zeta}$ multiplied by powers of $\bar{\zeta}\zeta$, ensuring that it obeys some self-duality constraint. The (self-dual) Higgs superfield Φ is to be handled in the normal way and will therefore couple the left and right fields together.

2.1 Expansion in two properties

Thus our starting point is the pair of properties, ζ^0 signifying neutrinicity and ζ^4 signifying charge -1 leptonicity, and a complete disregard of strong interactions. (The full $\text{Sp}(10)$ theory includes strong interactions via three further properties $\zeta^{1,2,3}$ signifying colour down-type quarks.) The anti-selfdual fermions' superfield and adjoint, properly normalized and simplified, consists of a single generation,

$$\begin{aligned}\bar{\Psi} &= [\bar{\zeta}_0(1 - \bar{\zeta}_4\zeta^4)\nu + \bar{\zeta}_4(1 - \bar{\zeta}_0\zeta^0)\ell]/\sqrt{2}, \\ \bar{\Psi} &\equiv [\bar{\nu}(1 - \bar{\zeta}_4\zeta^4)\zeta^0 + \bar{\ell}(1 - \bar{\zeta}_0\zeta^0)\zeta^4]/\sqrt{2}\end{aligned}\tag{1}$$

while the anti-selfdual Higgs superfield,

$$\Phi = Y(1 - \bar{\zeta}_0\zeta^0\bar{\zeta}_4\zeta^4)/\sqrt{2} + Z^0(\bar{\zeta}_0\zeta^0 - \bar{\zeta}_4\zeta^4)/\sqrt{2} + Z^+(\bar{\zeta}_0\zeta^4) + Z^-(\bar{\zeta}_4\zeta^0),\tag{2}$$

contains a weak isosinglet (Y) and isotriplet (Z^+, Z^0, Z^-). In that respect it is badly in conflict with the standard model. However bear in mind that the inclusion of the strong chromicity triplet fixes this problem and cures the lack of insufficient generations. The purpose of the present section is not to describe a totally realistic scenario but to comprehend how the masses of fermions and bosons are linked to the parameters arising from a renormalizable superHiggs potential comprising *two* sets of Higgs component fields Y and Z *interacting with one another* and with the fermions by means of a *single* Yukawa coupling constant h . [In the following section we do include some semblance of strong interactions, involving several Higgs component fields and thereby cure *some* of the deficiencies of this section.]

The fermionic Lagrangian arises from the usual combination

$$\begin{aligned} \mathcal{L}_\Psi &= \int d^2\bar{\zeta} d^2\zeta \bar{\Psi}(i\gamma.\partial - \sqrt{2}h\Phi)\Psi \\ &= \bar{\nu}[i\gamma.\partial - h(Y + Z^0/2)]\nu + \bar{\ell}[i\gamma.\partial - h(Y - Z^0/2)]\ell - h(Z^+\bar{\nu}\ell + Z^-\bar{\ell}\nu)/\sqrt{2} \quad (3) \end{aligned}$$

while the renormalizable scalar potential consists of up-to-quartic terms ²:

$$\begin{aligned} \mathcal{L}_\Phi &= - \int d^2\bar{\zeta} d^2\zeta [(\partial\Phi)^2/2 + \mu^2\Phi^2/2 + \sqrt{2}f\Phi^3/3 + g\Phi^4/6] \\ &= [(\partial Y)^2 + (\partial Z^0)^2]/2 + \partial Z^+.\partial Z^- + \mu^2[Y^2/2 + Z^{02}/2 + Z^+Z^-] \\ &\quad - f[Y^3/2 + YZ^{02} + 2YZ^+Z^-] - g[Y^4/6 + Y^2Z^{02}/2 + Y^2Z^+Z^-](4) \end{aligned}$$

2.2 Expectation values

There are five equations of motion: for the one charged and two uncharged Higgs fields and the two leptons. Minimising the scalar potential in (4) so that the two uncharged fields Y and Z^0 acquire expectation values,

$$Y = \langle Y \rangle + \mathcal{Y} \equiv y + \mathcal{Y}, \quad \text{and} \quad Z^0 = \langle Z^0 \rangle + \mathcal{Z} \equiv z + \mathcal{Z},$$

we arrive at two constraints. With just the classical quartic term, as in (4) the following conditions have to hold exactly:

$$f(3y^2/2 + z^2) + g(2y^3/3 + yz^2) - \mu^2y = 0, \quad 2fyz + gy^2z - \mu^2z = 0. \quad (5)$$

²The negative sign in front is an artifact of the fermionic integration and is really of no fundamental consequence; the correct signs appear in eq (4)

At the same time the mass² matrix for the \mathcal{Y} and \mathcal{Z} fields reads

$$\begin{pmatrix} -\mu^2 + 3fy + 2gy^2 + gz^2 & 2z(f + gy) \\ 2z(f + gy) & -\mu^2 + 2fy + gy^2 \end{pmatrix} \quad (6)$$

and we need to ensure the absence of tachyons. If we strictly try to solve eqs. (5) first there are two possibilities: (a) that $z = 0$ and $y \neq 0$ with $\mu^2 = 3fy/2 + 2gy^2/3$, whereupon $m_{\mathcal{Y}}^2 = 3fy/2 + 4gy^2/3$, $m_{\mathcal{Z}}^2 = fy/2 + gy^3/3$ can both be positive – so no tachyons – or (b) $z \neq 0$ and $2fy + gy^2 = \mu^2$, which case unfortunately leads to tachyonic \mathcal{Y} and \mathcal{Z} fields and is therefore unacceptable physically. Case (a) is perfectly fine physically but leads to degenerate neutrino and lepton fields, so is uninteresting.

2.3 Masses

The conclusions above signify that really we should not limit ourselves to a classical quartic potential. Indeed we know that the effective potential will include higher order terms in the participating fields from quantum loops; thus we shall relax the over-constrained system equations (5) and assume that they are only approximately true. This will allow us to escape the straight-jacket of degenerate fermion masses which otherwise ensue for we need two distinct nonzero values for y and z in (3). We will therefore continue to assume that $z \neq 0$ and introduce a small parameter Δ having dimension of mass² into the second equation (5) that is simply meant to encapsulate the result of higher order quantum corrections: $\mu^2 = 2fy + gy^2 + \Delta$. This proves sufficient to lead to a positive value for the determinant of the $(\mathcal{Y}, \mathcal{Z})$ mass² matrix, namely, $\Delta(\Delta + fy + gy^2 + \Delta) - 4z^2(f + gy)^2$; further it suggests that z is of order $\sqrt{\Delta}$. No longer are the charged lepton and neutrino mass degenerate. and all the results make physical sense. We shall adopt a similar strategy in the next section.

3 Expansion in three properties

There is much more substance in this section. To the pair $\zeta^{0,4}$ of leptonic properties which encapsulate weak isospin we shall attach a third (charged co-ordinate) ζ^5 which mimics strong interactions but without colour and which can be likened to the colourless product of (anti)red/blue/green down-quarks;

it carries fermion number $F = -1$ and charge $Q = 1$ ³. The table below summarises the quantum number attributes:

Property	Q	T_3	L	F
ζ^0	0	1/2	1	1
ζ^4	-1	-1/2	1	1
ζ^5	1	0	0	-1

Table 1: Charge Q , weak isospin T_3 , lepton number L and fermion number F of the three fundamental properties.

Since a superfield is to be expanded in powers of the three ζ and three $\bar{\zeta}$ we can come across combinations like $\bar{\zeta}_0\bar{\zeta}_4\zeta^5$ having $F = 3, Q = -2$ or $\bar{\zeta}_0\bar{\zeta}_5\zeta^4$ having $F = -1, Q = 2$; these are unpleasant states which have never been observed experimentally, so we need to find a way of excising them. This can be done by requiring the superfield to be anti-selfdual, associated with reflection about the cross-diagonal as explained in reference [3]. For instance the action of the duality operation (denoted by $^\times$) on these combinations is

$$(\bar{\zeta}_0\bar{\zeta}_4\zeta^5)^\times = \bar{\zeta}_0\bar{\zeta}_4\zeta^5, \quad (\bar{\zeta}_0\bar{\zeta}_5\zeta^4)^\times = \bar{\zeta}_0\bar{\zeta}_5\zeta^4.$$

This does not remove all exotic states, for instance the leptobaryon combinations,

$$\zeta^0\zeta^4 \text{ with } Q = -1, \quad \bar{\zeta}_4\zeta^5 \text{ with } Q = 2,$$

survive, etc. Upon imposing antiduality on both the superfields, Φ for bosons and Ψ for fermions, we arrive at expansions that comprise *two* generations of fermions (each consisting of a neutrino, charged lepton and ‘proton’) and *five* uncharged Higgs fields (three weak isosinglets A, B, U , one isotriplet C^0 and one isodoublet D^0) plus a few charged partners. Properly normalized, the antidual expansions read in full:

$$\begin{aligned} \sqrt{2}\Psi = & \bar{\zeta}_0\nu(1 + \bar{\zeta}_4\zeta^4\bar{\zeta}_5\zeta^5) + \bar{\zeta}_0\nu'(\bar{\zeta}_4\zeta^4 + \bar{\zeta}_5\zeta^5) \\ & + \bar{\zeta}_4\ell(1 + \bar{\zeta}_0\zeta^0\bar{\zeta}_5\zeta^5) + \bar{\zeta}_4\ell'(\bar{\zeta}_0\zeta^0 + \bar{\zeta}_5\zeta^5) \\ & + p^c\zeta^5(1 + \bar{\zeta}_0\zeta^0\bar{\zeta}_4\zeta^4) + p^{c'}\zeta^5(\bar{\zeta}_0\zeta^0 + \bar{\zeta}_4\zeta^4), \end{aligned} \quad (7)$$

³In previous papers we reserved the labels $\zeta^{1,2,3}$ for colour and shall continue to do so here, thereby ignoring them.

$$\begin{aligned}
\sqrt{2}\bar{\Phi} = & A(1 - \bar{\zeta}_0\zeta^0\bar{\zeta}_4\zeta^4\bar{\zeta}_5\zeta^5) + U(\bar{\zeta}_5\zeta^5 - \bar{\zeta}_4\zeta^4\bar{\zeta}_0\zeta^0) \\
& + B(\bar{\zeta}_0\zeta^0 + \bar{\zeta}_4\zeta^4)(1 - \bar{\zeta}_5\zeta^5)/\sqrt{2} \\
& + (C^+\bar{\zeta}_0\zeta^4 + C^-\bar{\zeta}_4\zeta^0 + C^0(\bar{\zeta}_0\zeta^0 - \bar{\zeta}_4\zeta^4)/\sqrt{2})(1 + \bar{\zeta}_5\zeta^5) \\
& + (D^0\zeta^4\zeta^5 - \bar{D}^0\bar{\zeta}_5\bar{\zeta}_4)(1 + \bar{\zeta}_0\zeta^0) + (D^-\zeta^0\zeta^5 + D^+\bar{\zeta}_0\bar{\zeta}_5)(1 + \bar{\zeta}_4\zeta^4) \\
& + (S^+\zeta^0\zeta^4 + S^-\bar{\zeta}_0\bar{\zeta}_4)(1 + \bar{\zeta}_5\zeta^5) + (T^{--}\bar{\zeta}_4\zeta^5 + T^{++}\bar{\zeta}_5\zeta^4)(1 + \bar{\zeta}_0\zeta^0) \\
& + (T^-\bar{\zeta}_0\zeta^5 + T^+\bar{\zeta}_5\zeta^0)(1 + \bar{\zeta}_4\zeta^4)
\end{aligned} \tag{8}$$

Thus we encounter three uncharged isosinglet Higgs A, B, U plus one charged combination S^+ ; two isospin doublets $(D^+, D^0), (T^{++}, T^+)$ and one isotriplet (C^+, C^-, C^0) . We can regard D and T as leptoquark fields; note that only the expectation value of the uncharged D^0 has a bearing on lepton-baryon mass splitting.

Expanding the various parts of the renormalizable Lagrangian for the superHiggs field purely on its own, viz.

$$\mathcal{L} = \int d^3\bar{\zeta} d^3\zeta [(\partial\Phi)^2/2 + \mu^2\Phi^2/2 - \sqrt{2}f\Phi^3/3 - g\Phi^4/4], \tag{9}$$

we encounter the following complicated combinations of terms::

$$\begin{aligned}
\int d^3\bar{\zeta} d^3\zeta \Phi^2 = & A^2 + B^2 + U^2 + C^{02} + 2C^+C^- + \\
& 2(D^+D^- + \bar{D}^0D^0 + S^+S^- + T^+T^- + T^{++}T^{--});
\end{aligned} \tag{10}$$

$$\begin{aligned}
\sqrt{2} \int d^3\bar{\zeta} d^3\zeta \Phi^3/3 = & A[A^2/2 + U^2 + B^2 + C^{02} + 2C^+C^- + \\
& 2\bar{D}^0D^0 + 2D^+D^- + 2S^+S^- + 2T^+T^- + 2T^{++}T^{--}] + \\
& U(C^+C^- + C^{02}/2 - B^2/2 + S^+S^-) + \\
& B(\bar{D}^0D^0 + D^+D^- + T^{++}T^{--} + T^+T^-)/\sqrt{2} + \\
& C^0(\bar{D}^0D^0 - D^+D^- + T^{++}T^{--} - T^+T^-)/\sqrt{2} + \\
& S^+(\bar{D}^0T^- - D^+T^{--}) + S^-(D^0T^+ - D^-T^{++}) - \\
& C^+(\bar{D}^0D^- + T^+T^{--}) - C^-(D^0T^+ + T^-T^{++});
\end{aligned} \tag{11}$$

$$\begin{aligned}
\int d^3\bar{\zeta} d^3\zeta \Phi^4/6 = & A^2[A^2/6 + (B^2 + U^2 + C^{02})/2 + C^+C^- + \\
& \bar{D}^0D^0 + D^+D^- + S^+S^- + T^+T^- + T^{++}T^{--}] + \\
& AU(C^+C^- + C^{02}/2 - B^2/2 + S^+S^-) +
\end{aligned}$$

$$\begin{aligned}
& AB(\bar{D}^0 D^0 + D^+ D^- + T^{++} T^{--} + T^+ T^-)/\sqrt{2} + \\
& AC^0(\bar{D}^0 D^0 - D^+ D^- + T^{++} T^{--} - T^+ T^-)/\sqrt{2} + \\
& AS^+(\bar{D}^0 T^- - D^+ T^{--}) + AS^-(D^0 T^+ - D^- T^{++}) - \\
& AC^+(\bar{D}^0 D^- + T^+ T^{--}) - AC^-(D^0 T^+ + T^- T^{++}). \quad (12)
\end{aligned}$$

As we are looking for spontaneously broken solutions which are nevertheless charge-conserving, the only expectation values we can allow belong to the fields A, B, U, C^0, D^0 . We therefore make the expansions

$$A = \langle A \rangle + \mathcal{A} \equiv a + \mathcal{A}, \quad B = \langle B \rangle + \mathcal{B} \equiv b + \mathcal{B}, \quad U = \langle U \rangle + \mathcal{U} \equiv u + \mathcal{U},$$

$$C^0 = \langle C^0 \rangle + \mathcal{C} \equiv c + \mathcal{C}, \quad D^0 = \langle D^0 \rangle + \mathcal{D} \equiv d/\sqrt{2} + \mathcal{D},$$

and ascertain the minimum of the quartic potential; this occurs where

$$\begin{aligned}
\mu^2 a &= f[3a^2/2 + b^2 + u^2 + c^2 + d^2] + \\
&g[a(2a^2/3 + b^2 + c^2 + d^2 + u^2) + u(c^2 - b^2)/2 + d^2(b + c)/2\sqrt{2}](13)
\end{aligned}$$

$$\mu^2 b = f[2ab - ub + d^2/2\sqrt{2}] + g[ab(a - u) + ad^2/2\sqrt{2}], \quad (14)$$

$$\mu^2 u = f[2au + (c^2 - b^2)/2] + g[a^2 u + a(c^2 - b^2)/2], \quad (15)$$

$$\mu^2 c = f[2ac + uc + d^2/2\sqrt{2}] + g[ac(a + u) + ad^2/2\sqrt{2}], \quad (16)$$

$$\mu^2 d = f[2ad + d(b + c)/\sqrt{2}] + g[a^2 d + ad(b + c)/\sqrt{2}]. \quad (17)$$

The following combinations are obiquitous so we shall abbreviate them for future use:

$$F \equiv f + ga, \quad G \equiv 2fa + ga^2.$$

Firstly let us study the fermion mass matrix hailing from the interaction $\int d^3\bar{\zeta} d^3\zeta \bar{\Psi}[m + \sqrt{2}h\langle\Phi\rangle]\Psi$. In the space of states $(\nu, \nu', \ell, \ell', p^c, p^{c'})$, we obtain the elements:

$$h \begin{pmatrix} \alpha - \frac{b+c}{2\sqrt{2}} & \frac{u}{2} + \frac{b-c}{2\sqrt{2}} & 0 & 0 & 0 & 0 \\ \frac{u}{2} + \frac{b-c}{2\sqrt{2}} & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha - \frac{b-c}{2\sqrt{2}} & \frac{u}{2} + \frac{b+c}{2\sqrt{2}} & -\frac{d}{2\sqrt{2}} & -\frac{d}{2\sqrt{2}} \\ 0 & 0 & \frac{u}{2} + \frac{b+c}{2\sqrt{2}} & \alpha & -\frac{d}{2\sqrt{2}} & 0 \\ 0 & 0 & -\frac{d}{2\sqrt{2}} & -\frac{d}{2\sqrt{2}} & \alpha - \frac{u}{2} & \frac{b}{\sqrt{2}} \\ 0 & 0 & -\frac{d}{2\sqrt{2}} & 0 & \frac{b}{\sqrt{2}} & \alpha \end{pmatrix}, \quad (18)$$

where $\alpha = a + m/h$. Thus it is essential to look for solutions where $d \neq 0$ in order to separate the lepton and baryon sectors, and $u + b - c \neq 0$ if we wish to separate generations.

Next let us examine the mass matrices of the various scalar mesons, which must conform *approximately* to the above restrictions on the expectation values since we know that the effective potential will disturb conditions (13) to (17). It is important for us to ensure there are no tachyonic particles. Start with the Higgs multiplets contained in Φ , of which the only unmixed state is T^{++} . It has

$$M_{T^{++}}^2 = G + (b + c)F/\sqrt{2} - \mu^2. \quad (19)$$

On the other hand the states $(C^+, -D^+)$ and separately (S^+, T^+) mix with identical mass² matrices and therefore form degenerate pairs:

$$\begin{pmatrix} G + uF - \mu^2 & dF/\sqrt{2} \\ dF/\sqrt{2} & G + (b - c)F\sqrt{2} - \mu^2 \end{pmatrix}. \quad (20)$$

Next, the full set of (symmetric) 5×5 matrix elements for the uncharged quantum states $(A, B, C^0, U, (D^0 + \bar{D}^0)/\sqrt{2})$ read:

$$M_{AA}^2 = aF + G + g(b^2 + c^2 + u^2 + d^2) - \mu^2, \quad (21)$$

$$M_{AB}^2 = M_{BA}^2 = 2bF - gbu + gd^2/2\sqrt{2}, \quad (22)$$

$$M_{AC}^2 = M_{CA}^2 = 2cF + gcu + gd^2/2\sqrt{2}, \quad (23)$$

$$M_{AU}^2 = M_{UA}^2 = 2uF + g(c^2 - b^2)/2, \quad (24)$$

$$M_{AD}^2 = M_{DA}^2 = 2dF + gd(b + c)/\sqrt{2}, \quad (25)$$

$$M_{BB}^2 = G - uF - \mu^2, \quad M_{BC}^2 = M_{CB}^2 = 0, \quad (26)$$

$$M_{BU}^2 = M_{UB}^2 = -bF, \quad M_{BD}^2 = M_{DB}^2 = dF/\sqrt{2}, \quad (27)$$

$$M_{CC}^2 = G + uF - \mu^2, \quad (28)$$

$$M_{CU}^2 = M_{UC}^2 = cF, \quad M_{CD}^2 = M_{DC}^2 = dF/\sqrt{2}, \quad (29)$$

$$M_{UU}^2 = G - \mu^2, \quad M_{UD}^2 = M_{DU}^2 = 0, \quad (30)$$

$$M_{DD}^2 = G + (b + c)F/\sqrt{2} - \mu^2. \quad (31)$$

The most practical strategy for analysing the remaining conditions which ensure that we get physically acceptable solutions is as follows. There are two free parameters in our scheme (apart from coupling constants and masses) that are *not* determined by minimising the effective potential and these

amount to fixing somehow two of the five expectation values of the super-Higgs components – which must therefore be found by other considerations, such as dynamical symmetry breaking or the inclusion of quantum corrections. To gain an idea first of what is possible, consider the particular case where $b = c$ and $u = 0$. One then readily checks that no tachyon states arise provided the conditions below are met:

$$G - \mu^2 \geq \sqrt{2}|bF| \quad (32)$$

$$G - \mu^2 \geq |dF|/\sqrt{2}. \quad (33)$$

and the eigenvalues of the 3×3 mass² for the coupled fields $A, (B+C^0), (D^0 + \bar{D}^0)$

$$\begin{pmatrix} aF + G + g(2b^2 + d^2) - \mu^2 & 2bF + gd^2/2\sqrt{2} & 2d(F + gb/\sqrt{2}) \\ 2bF + gd^2/2\sqrt{2} & G - \mu^2 & dF/\sqrt{2} \\ 2d(F + gb/\sqrt{2}) & dF/\sqrt{2} & G + \sqrt{2}bF - \mu^2 \end{pmatrix}$$

are positive; and this is indeed possible. Inclusion of additional corrections arising when $b \neq c$ and $u \neq 0$ does not disturb the conclusion that there are no tachyons. The last consideration, which is the least important, is how well the minimisation conditions of the classical quartic potential, eqs (13)-(17) are satisfied, since we are certain that they are liable to be muddled by quantum loops⁴. For purposes of illustration, we can show that ‘acceptable’ masses can arise for some suitable inputs (masses in GeV for dimensionful quantities):

Dimensionless couplings : $g = 2, h = 1$; Dimensionful coupling : $f = 0.3$;

Masses : $m = 0$; $-\mu^2 = 1.2$;

Expectation values : $a = 0.5, b = c = 0.5, u = -0.5, d = -0.3$.

The consequence of these rough choices is the following sets of particle masses for fermions and Higgs mesons:

Neutrinos : $m_\nu = 0.02, m'_\nu = 0.63$;

Leptons/Baryons : $m_\ell = 0.30, m'_\ell = 0.44, m_p = 0.53, m'_p = 0.98$;

⁴If the constraints (13) to(17) are strictly imposed we arrive at the ridiculous result that there are no tachyons but all particles are massless

Doubly charged Higgs meson : $m_{T^{++}}^2 = 4.35$;

Singly charged Higgs mesons : $m_{C^+}^2 = m_{S^+}^2 = 1.78$, $m_{D^+}^2 = m_{T^+}^2 = 3.09$;

Uncharged Higgs : $m_A^2 = 9.00$, $m_D^2 = 3.01$, $m_C^2 = 4.35$, $m_U^2 = 2.34$, $m_B^2 = 0.09$,

which values can be scaled up and adjusted further if needed by varying the inputs. The small deviations from the classical quartic potential constraints can thereby be estimated. For instance, typically $\Delta \sim u(G - \mu^2) \simeq 2.9$ enters eq (15) and corresponds to the magnitude expected from quantum effects.

These results are not ridiculous for a very crude 2-generation model. They give hope that a more realistic model based on 5 property coordinates might produce reasonable values for the known generations of colourless particles. But in any case schemes of this type have fewer parameters than the standard model; thus the masses and mixings of fermions in conventional model contain at least 18 independent constants ignoring CP violation, whereas 5-property schemes only contain at most 9 Higgs expectation values and only *one* Yukawa coupling. The latter are therefore worth exploring. Also of great interest is to what extent the initial anti-selfduality symmetry of the superfields is preserved by quantum corrections. This can be verified by looking at the one-loop corrections to the self-energies of the participating fields and the 3-point Yukawa couplings to see if they are renormalized by exactly the same amount; but this is a major study and must be left to a future investigation.

Appendix A - Simplifying treatment of the left/right fields

Let us focus on a single complex ζ , corresponding to an $\text{Sp}(2)$ group structure, associated with a single fermion number. Strictly, the fermion superfield $\Psi(x, \zeta, \bar{\zeta})$ must depend on property and antiproperty and also involve both left and right chirality components. Recalling the usual charge conjugation relation, $\psi^c = C\bar{\psi}$, we note the basic result, $\psi_L^c = \psi_R^c$. It follows that if the source superfield is considered to be totally left-handed,

$$\Psi(x, \zeta, \bar{\zeta}) = \bar{\zeta}\psi_L + \psi_L^c\zeta,$$

it does in reality incorporate both chirality components. The superfield adjoint then has to be defined as

$$\bar{\Psi}(x, \zeta, \bar{\zeta}) = \bar{\psi}_R\zeta - \bar{\zeta}\bar{\psi}_R^c,$$

in order to ensure

$$\bar{\Psi}\Psi = \zeta\bar{\zeta} [\bar{\psi}_R\psi_L + \bar{\psi}_R^c\psi_L^c] = \zeta\bar{\zeta} [\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R],$$

in tandem with the Grassmannian integration rule: $\int d\bar{\zeta} d\zeta \zeta\bar{\zeta} = 1$.

Bearing in mind that Ψ is overall *Bose*, the kinetic energy term needs to be taken as the bilinear combination,

$$\Psi_\alpha (iC^{-1}\gamma.\partial)^{\alpha\beta} (\zeta\partial_\zeta - \bar{\zeta}\partial_{\bar{\zeta}})\Psi_\beta = \zeta\bar{\zeta} [\bar{\psi}_L i\gamma.\partial\psi_L + \bar{\psi}_L^c i\gamma.\partial\psi_L^c] = \zeta\bar{\zeta} [\bar{\psi}_L i\gamma.\partial\psi_L + \bar{\psi}_R i\gamma.\partial\psi_R].$$

These manoeuvres are of great importance if one is interested in gauging one or other of the chiralities. For instance a local left-field phasing would engender the gauge combination $A\bar{\zeta}\partial_{\bar{\zeta}}$ *only*, while the right-field is accompanied by $A\zeta\partial_\zeta$, etc. Turning to the superfield scalar Φ , which couples left with right, it is simply described by the the self-dual combination

$$\Phi(x, \zeta, \bar{\zeta}) = [1 - \bar{\zeta}\zeta]H/\sqrt{2}$$

so the Yukawa interaction with the source superfield is nothing but the usual interaction:

$$\sqrt{2}h \int d\bar{\zeta} d\zeta \bar{\Psi}\Phi\Psi = h \int d\bar{\zeta} d\zeta (\zeta\bar{\zeta}) [\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R](1 - \zeta\bar{\zeta})H = h\bar{\psi}\psi H/\sqrt{2}.$$

Upon including the kinetic term of the source and adding that of the scalar field, namely

$$\int d\bar{\zeta} d\zeta (\partial\Phi)^2/2 = (\partial H)^2/2,$$

plus the renormalizable self-interaction

$$\int d\bar{\zeta} d\zeta [\mu^2\Phi^2/2 - \lambda\Phi^4/4] = \mu^2 H^2/2 - \lambda H^4/4,$$

we are back to the original Higgs model.

This may seem like a sledgehammer to crack a nut (and it probably is) but, when we add further properties, exciting new possibilities arise. As we are only interested in the mass generation mechanism through coupling to the scalar superfield and are not concerned about the gauge field, we can bypass the left-right fuss by combining the components into the Dirac superfields $\Psi = \bar{\zeta}\psi$, $\bar{\Psi} = \bar{\psi}\zeta$ and just leave it at that, apart from products with polynomials of $\bar{\zeta}\zeta$ which can occur with more than one property when one is imposing self-duality on the superfields. That is the strategy we have adopted in sections 2 and 3.

Acknowledgements

I would like to express my thanks to Peter Jarvis and Paul Stack for numerous discussions and assistance.

References

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